



National
Qualifications
2016

2016 Mathematics Paper 1 (Non-calculator)

National 5

Finalised Marking Instructions

© Scottish Qualifications Authority 2016

The information in this publication may be reproduced to support SQA qualifications only on a non-commercial basis. If it is to be used for any other purposes written permission must be obtained from SQA's NQ Assessment team.

Where the publication includes materials from sources other than SQA (secondary copyright), this material should only be reproduced for the purposes of examination or assessment. If it needs to be reproduced for any other purpose it is the centre's responsibility to obtain the necessary copyright clearance. SQA's NQ Assessment team may be able to direct you to the secondary sources.

These Marking Instructions have been prepared by Examination Teams for use by SQA Appointed Markers when marking External Course Assessments. This publication must not be reproduced for commercial or trade purposes.



General Marking Principles for National 5 Mathematics

This information is provided to help you understand the general principles you must apply when marking candidate responses to questions in this Paper. These principles must be read in conjunction with the detailed marking instructions, which identify the key features required in candidate responses.

For each question the marking instructions are generally in two sections, namely Illustrative Scheme and Generic Scheme. The Illustrative Scheme covers methods which are commonly seen throughout the marking. The Generic Scheme indicates the rationale for which each mark is awarded. In general, markers should use the Illustrative Scheme and only use the Generic Scheme where a candidate has used a method not covered in the Illustrative Scheme.

- (a) Marks for each candidate response must always be assigned in line with these General Marking Principles and the Detailed Marking Instructions for this assessment.
- (b) Marking should always be positive. This means that, for each candidate response, marks are accumulated for the demonstration of relevant skills, knowledge and understanding: they are not deducted from a maximum on the basis of errors or omissions.
- (c) If a specific candidate response does not seem to be covered by either the principles or detailed Marking Instructions, and you are uncertain how to assess it, you must seek guidance from your Team Leader.
- (d) Credit must be assigned in accordance with the specific assessment guidelines.
- (e) One mark is available for each •. There are no half marks.
- (f) Working subsequent to an error must be **followed through**, with possible credit for the subsequent working, provided that the level of difficulty involved is approximately similar. Where, subsequent to an error, the working for a follow through mark has been eased, the follow through mark cannot be awarded.
- (g) As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Unless specifically mentioned in the marking instructions, a correct answer with no working receives no credit.
- (h) Candidates may use any mathematically correct method to answer questions except in cases where a particular method is specified or excluded.
- (i) As a consequence of an error perceived to be trivial, casual or insignificant, eg $6 \times 6 = 12$ candidates lose the opportunity of gaining a mark. However, note the second example in comment (j).

- (j) Where a transcription error (paper to script or within script) occurs, the candidate should normally lose the opportunity to be awarded the next process mark, eg

This is a transcription error and so the mark is not awarded.		$x^2 + 5x + 7 = 9x + 4$
Eased as no longer a solution of a quadratic equation so mark is not awarded.		$x - 4x + 3 = 0$
Exceptionally this error is not treated as a transcription error as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.		$x = 1$
		$x^2 + 5x + 7 = 9x + 4$
		$x - 4x + 3 = 0$
		$(x - 3)(x - 1) = 0$
		$x = 1 \text{ or } 3$

(k) **Horizontal/vertical marking**

Where a question results in two pairs of solutions, this technique should be applied, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bullet^5$		$\bullet^6$	
$\bullet^5$	$x = 2$		$x = -4$	
$\bullet^6$	$y = 5$		$y = -7$	

Horizontal: $\bullet^5 x = 2$ and $x = -4$ Vertical: $\bullet^5 x = 2$ and $y = 5$
 $\bullet^6 y = 5$ and $y = -7$ $\bullet^6 x = -4$ and $y = -7$

Markers should choose whichever method benefits the candidate, but **not** a combination of both.

- (l) In final answers, unless specifically mentioned in the detailed marking instructions, numerical values should be simplified as far as possible, eg:

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\frac{15}{0.3}$ must be simplified to 50 $\sqrt{64}$ must be simplified to 8*	$\frac{43}{1}$ must be simplified to 43 $\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$
--	---

*The square root of perfect squares up to and including 100 must be known.

- (m) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(n) Unless specifically mentioned in the marking instructions, the following should not be penalised:

- Working subsequent to a correct answer
- Correct working in the wrong part of a question
- Legitimate variations in numerical answers/algebraic expressions, eg angles in degrees rounded to nearest degree
- Omission of units
- Bad form (bad form only becomes bad form if subsequent working is correct), eg $(x^3 + 2x^2 + 3x + 2)(2x + 1)$ written as $(x^3 + 2x^2 + 3x + 2) \times 2x + 1$

$2x^4 + 4x^3 + 6x^2 + 4x + x^3 + 2x^2 + 3x + 2$ written as $2x^4 + 5x^3 + 8x^2 + 7x + 2$ gains full credit

- Repeated error within a question, but not between questions or papers
- (o) In any 'Show that...' question, where the candidate has to arrive at a required result, the last mark of that part is not available as a follow-through from a previous error unless specified in the detailed marking instructions.
- (p) All working should be carefully checked, even where a fundamental misunderstanding is apparent early in the candidate's response. Marks may still be available later in the question so reference must be made continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that the candidate has gained all the available marks.
- (q) Scored-out working which has not been replaced should be marked where still legible. However, if the scored out working has been replaced, only the work which has not been scored out should be marked.
- (r) Where a candidate has made multiple attempts using the same strategy and not identified their final answer, mark all attempts and award the lowest mark. Where a candidate has tried different valid strategies, apply the above ruling to attempts within each strategy and then award the highest resultant mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Detailed Marking Instructions for each question

Question			Generic Scheme	Illustrative Scheme	Max Mark
1.			Ans: $\begin{pmatrix} -3 \\ -4 \end{pmatrix}$ •¹ calculate $\frac{1}{2}p$ •² solution	•¹ $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ •² $\begin{pmatrix} -3 \\ -4 \end{pmatrix}$	2
Notes: - Correct answer without working award 2/2 - Treat $\begin{pmatrix} -3 \\ -4 \end{pmatrix}$ as bad form award 2/2 - Where there are no brackets ie $\begin{matrix} -3 \\ -4 \end{matrix}$ award 1/2 - For $\frac{-3}{-4}$ award 1/2 - Where there is invalid subsequent working •² is not available eg award 1/2 ✓× for the following: $\begin{pmatrix} -3 \\ -4 \end{pmatrix} \rightarrow$ (a) $(-3, -4)$ (b) $-3 + (-4) = -7$ (c) $\sqrt{(-3)^2 + (-4)^2} = 5$					
Commonly Observed Responses: $\begin{pmatrix} 4 \\ -6 \end{pmatrix} + \begin{pmatrix} -5 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ -7 \end{pmatrix}$ award 1/2 $\frac{1}{2} \left(\begin{pmatrix} 4 \\ -6 \end{pmatrix} + \begin{pmatrix} -5 \\ -1 \end{pmatrix} \right) = \begin{pmatrix} -0.5 \\ -3.5 \end{pmatrix}$ award 1/2 $\begin{pmatrix} 4 \\ -6 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} -5 \\ -1 \end{pmatrix} = \begin{pmatrix} 1.5 \\ -6.5 \end{pmatrix}$ award 1/2					

Question			Generic Scheme	Illustrative Scheme	Max Mark
2.			Ans: $\frac{13}{28}$ Method 1 •¹ start the calculation correctly •² consistent answer in simplest form Method 2 •¹ start the calculation correctly •² consistent answer in simplest form	•¹ $\frac{7}{21} + \frac{6}{21}$ •² $\frac{13}{28}$ •¹ $\frac{3}{12} + \frac{6}{28}$ or equivalent •² $\frac{13}{28}$	2
Notes: - Correct answer without working award 0/2. - Final answer must be in simplest form eg for $\frac{39}{84}$ award 1/2 ✓× •² is only available where simplifying is required. - For subsequent incorrect working, •² is not available eg for $\frac{13}{28} = 2\frac{2}{28} = 2\frac{1}{14}$ award 1/2 ✓×					
Commonly Observed Responses: For an answer of $\frac{9}{40}$ obtained from (a) Method 1: $\frac{3}{4}\left(\frac{1}{3} + \frac{2}{7}\right) = \frac{3}{4} \times \frac{3}{10} = \frac{9}{40}$ award 0/2 (b) Method 2: $\frac{3}{12} + \frac{6}{28} = \frac{9}{40}$ award 1/2 ✓×					

Question			Generic Scheme	Illustrative Scheme	Max Mark
3.			Ans: 157 cm² Method 1 •¹ appropriate fraction •² correct substitution into area of sector formula •³ calculate area of sector Method 2 •¹ appropriate fraction •² correct substitution into area of sector formula •³ calculate area of sector	•¹ $\frac{45}{360}$ or equivalent •² $\frac{45}{360} \times 3 \cdot 14 \times 20^2$ •³ 157 (cm²) •¹ $\frac{360}{45}$ or equivalent •² $3 \cdot 14 \times 20^2 \div \frac{360}{45}$ •³ 157 (cm²)	3

Notes:

1. Correct answer without working award 0/3.
2. Accept “ $\div 8$ ” in working as evidence of $\frac{45}{360}$.
3. Accept “ $\times 3 \cdot 14$ ” in working as evidence of substitution into formula.

Commonly Observed Responses:

1. $\frac{45}{360} \times \pi r^2 = 8 \times 3 \cdot 14 \times 20^2 = 10048 (\text{cm}^2)$ award 2/3 ✓✓×
2. $\frac{360}{45} \times \pi r^2 = 8 \times 3 \cdot 14 \times 20^2 = 10048 (\text{cm}^2)$ award 2/3 ✓×✓
3. $\frac{45}{360} \times 3 \cdot 14 \times 20^2 \left(= \frac{45}{360} \times 3 \cdot 14 \times 40 \right) = 15 \cdot 7 (\text{cm}^2)$ award 2/3 ✓✓×
4. $\frac{45}{360} \times 3 \cdot 14 \times 40 = 15 \cdot 7 (\text{cm}^2)$ award 2/3 ✓×✓
5. $\frac{45}{360} \times \pi \times 20^2$ award 1/3 ✓××
6. $3 \cdot 14 \times 20^2 = 1256 (\text{cm}^2)$ award 0/

Question			Generic Scheme	Illustrative Scheme	Max Mark
4.	(a)		Ans: $2c + 3d = 9 \cdot 6$ • ¹ construct equation	• ¹ $2c + 3d = 9 \cdot 6$	1
Notes:					
Commonly Observed Responses:					
	(b)		Ans: $3c + 4d = 13 \cdot 3$ • ¹ construct equation	• ¹ $3c + 4d = 13 \cdot 3$	1
Notes:					
Commonly Observed Responses:					
	(c)		Ans: A cloak requires $1 \cdot 5 \text{ m}^2$ of material A dress requires $2 \cdot 2 \text{ m}^2$ of material • ¹ evidence of scaling • ² follow a valid strategy through to produce values for c and d • ³ calculate correct values for c and d • ⁴ communicate answers in square metres	• ¹ eg $6c + 9d = 28 \cdot 8$ $6c + 8d = 26 \cdot 6$ • ² values for c and d • ³ $c = 1 \cdot 5$ and $d = 2 \cdot 2$ • ⁴ cloak $1 \cdot 5 \text{ m}^2$ dress $2 \cdot 2 \text{ m}^2$	4
Notes: 1. Correct answer without working award 0/4. 2. • ⁴ is not available if either c or d is negative. 3. (a) where a candidate calculates values for c and d , • ⁴ can only be awarded for a conclusion containing the words 'cloak' and 'dress' along with the correct units in both cases (b) where a candidate only calculates a value for either c or d , • ⁴ can only be awarded if the conclusion contains the word 'cloak' or 'dress' along with the correct units					
Commonly Observed Responses:					

Question			Generic Scheme	Illustrative Scheme	Max Mark
5.	(a)		Ans: $W = 20A + 40$ •¹ gradient •² substitute gradient and a point into $y = mx + c$ or $y - b = m(x - a)$ •³ state equation in terms of W and A and in simplest form (remove any brackets and collect constants)	•¹ $\frac{240}{12}$ or equivalent •² $y - 100 = \frac{240}{12}(x - 3)$ or $y - 340 = \frac{240}{12}(x - 15)$ or $100 = \frac{240}{12} \times 3 + c$ or $340 = \frac{240}{12} \times 15 + c$ •³ $W = 20A + 40$ or equivalent	3

Notes:

- Correct answer without working award 3/3.
- ³ is not available for invalid subsequent working
eg $W = 20A + 40 \rightarrow W = 2A + 4$ award 2/3 ✓✓×

- Where $\frac{240}{12}$ is simplified incorrectly •² is still available

eg $m = \frac{240}{12} = \frac{20}{3} \rightarrow y - 100 = \frac{20}{3}(x - 3) \rightarrow W = \frac{20}{3}A + 80$ award 2/3 ✓✓×

Commonly Observed Responses:

- $y = 20x + 40$ award 2/3 ✓✓×
- $y = 20x$ award 1/3 ✓××
- $W = \frac{20}{1}A + 40$ award 2/3 ✓✓×
- $y - 100 = 20x - 3 \rightarrow W = 20A + 97$ award 2/3 ✓×✓

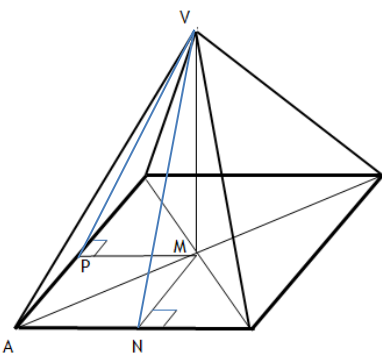
	(b)		Ans: $20 \times 12 + 40 = 280$ kg •¹ calculate weight using equation from part (a)	•¹ $20 \times 12 + 40 = 280$ (kg) stated explicitly	1
--	-----	--	--	---	---

Notes:

- Correct answer without working award 0/1
- Follow through mark from part (a) is only available if 12 is multiplied or divided by a whole number greater than 10 or a non-integer value followed by an addition or subtraction.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
6.			Ans: real and distinct •¹ find discriminant •² state nature of roots	•¹ 53 $[5^2 - 4 \times 7 \times (-1)]$ •² real and distinct (or equivalent)	2
Notes: 1. Correct answer without working award 0/2 2. $25 + 28 \rightarrow$ real and distinct award 2/2 3. eg $25 + 28 = 52 \rightarrow$ real and distinct award 1/2 $\times \checkmark$ 4. Accept 'real roots' 5. Do not accept 'two distinct roots' 6. Do not award •² where conclusion is ambiguous eg $53 \rightarrow$ roots are real and even award 1/2 $\checkmark \times$					
Commonly Observed Responses: 1. $\frac{-5 \pm \sqrt{5^2 - 4 \times 7 \times (-1)}}{2 \times 7} = \frac{-5 \pm \sqrt{53}}{2 \times 7}$ award 1/2 $\checkmark \times$ 2. $-3 \rightarrow$ no real roots award 1/2 $\times \checkmark$ 3. $-3 \rightarrow$ no roots award 0/2					

Question			Generic Scheme	Illustrative Scheme	Max Mark
7.	(a)		Ans: (8, 4, 0) • ¹ state coordinates of B	• ¹ (8, 4, 0)	1
Notes: 1. Brackets must be shown.					
Commonly Observed Responses:					
	(b)		Ans: 7 • ¹ know how to find AM^2 • ² know how to find AV • ³ find length of AV	• ¹ $3^2 + 2^2$ • ² $\sqrt{6^2 + (3^2 + 2^2)}$ • ³ 7	3
Notes: 1. Correct answer without working award 0/3 2. Alternative methods:					
					
(a)[know how to find AM^2] •¹ $\frac{1}{4}(6^2 + 4^2)$ •² $\sqrt{6^2 + \frac{1}{4}(6^2 + 4^2)}$ •³ 7 (b)[know how to find VN^2] •¹ $6^2 + 2^2$ •² $\sqrt{3^2 + (6^2 + 2^2)}$ •³ 7 (c)[know how to find VP^2] •¹ $6^2 + 3^2$ •² $\sqrt{2^2 + (6^2 + 3^2)}$ •³ 7					
Commonly Observed Responses:					
1. • ¹ $\begin{pmatrix} 3 \\ 2 \\ 6 \end{pmatrix} \rightarrow \bullet^2 \sqrt{3^2 + 2^2 + 6^2} \rightarrow \bullet^3 = 7$ award 3/3					
2. $\begin{pmatrix} 7 \\ 2 \\ 6 \end{pmatrix} \rightarrow \sqrt{7^2 + 2^2 + 6^2} = \sqrt{89}$ award 1/3 ✕✓✕					

Question			Generic Scheme	Illustrative Scheme	Max Mark
8.			Ans: $x = -\frac{5}{8}$ Method 1 •¹ multiply throughout by 6 •² rearrange •³ solve for x Method 2 •¹ rearrange •² start to solve for x •³ solve for x	•¹ $4x - 5 = 12x$ •² $-8x = 5$ or $-5 = 8x$ •³ $x = -\frac{5}{8}$ or $x = -0.625$ •¹ $\frac{4}{3}x = -\frac{5}{6}$ •² $x = -\frac{5}{6} \times \frac{3}{4}$ or $24x = -15$ or equivalent •³ $x = -\frac{5}{8}$ or $x = -0.625$	3

Notes:

1. Correct answer without working award 0/3
2. •¹ is available for multiplying throughout by any common multiple of 3 and 6
3. •¹ is not available for $\frac{4x-5}{6} = 2x$, $\frac{12x-15}{18} = 2x$ etc.
4. For the award of •³, the answer must be a non-integer value

Commonly Observed Responses:

9.			Ans: $\frac{2\sqrt{5}}{5}$ •¹ correct substitution •² consistent answer	•¹ $\frac{2}{\sqrt{5}}$ •² $\frac{2\sqrt{5}}{5}$	
----	--	--	---	---	--

Notes:

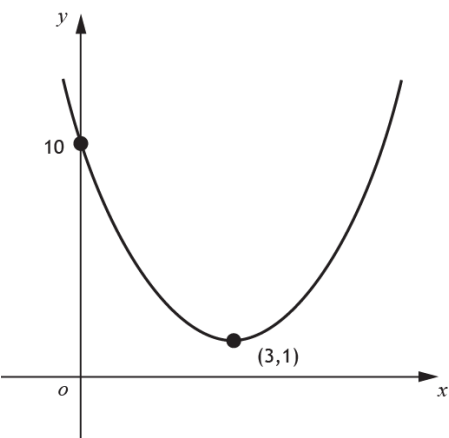
1. Correct answer without working award 0/2.

Commonly Observed Responses:

1. •² is not available where there is invalid subsequent working

eg $\frac{2\sqrt{5}}{5} = 2\sqrt{5}$ award 1/2 ✓×

2. $\frac{2}{\sqrt{x}} \times \frac{\sqrt{x}}{\sqrt{x}} = \frac{2\sqrt{x}}{x}$ award 1/2 ×✓

Question	Generic Scheme	Illustrative Scheme	Max Mark
10.	Ans:  •¹ coordinates of turning point correct •² sketch parabola with minimum turning point consistent with •¹ •³ y-intercept correct	•¹ (3,1) •² parabola with minimum turning point consistent with •¹ •³ (0,10) or 10	3
Notes: 1. Correct answer without working award 3/3. 2. Where the coordinates of the turning point are not stated elsewhere, then for a sketch of a parabola with minimum turning point (3,-1), (-3,±1) or (±1,±3) award •² but not •¹. Otherwise •² is only available where the minimum turning point indicated on the sketch is consistent with that stated elsewhere. 3. The sketch of the parabola need not meet or cut the y-axis for the award of •². 4. •² is not available if the parabola has a maximum turning point. 5. •³ is not available if the minimum turning point is on the y-axis. 6. Award •³ where the y-intercept is calculated to be at y=10 and is plotted on the diagram at (0,10) but annotated as (10,0). Treat this special case as bad form.			
Commonly Observed Responses:			

Question			Generic Scheme	Illustrative Scheme	Max Mark
11.			Ans: $\sin^2 x^\circ$ • ¹ identify correct trigonometric identity to be used • ² use correct trigonometric identity to simplify expression	• ¹ $\frac{\sin x}{\cos x}$ or $\frac{\sin^2 x}{\cos^2 x}$ • ² $\frac{\sin^2 x}{\cos^2 x} \times \cos^2 x = \sin^2 x$	2

Notes:

- $\sin^2 x$ without working award 0/2
- Degree signs are not required
- ² is not available if there is invalid subsequent working
eg (a) $\frac{\sin^2 x}{\cos^2 x} \times \cos^2 x = \sin^2 x = 1 - \cos x$ award 1/2 ✓×
(b) $\frac{\sin^2 x}{\cos^2 x} \times \cos^2 x = \sin^2 x = 1 - \cos^2 x$ award 2/2
- ¹ is not available if there are no variables e.g. $\frac{\sin^2}{\cos^2} \times \cos^2 = \sin^2$ award 1/2 ×✓
- ¹ is not available if candidate simply states $\tan x = \frac{\sin x}{\cos x}$ and $\sin^2 x + \cos^2 x = 1$ then proceeds no further
- Alternative acceptable strategies
 (a) •¹ $\tan x \cos x = \sin x$
 •² $\tan^2 x \cos^2 x = \sin^2 x$
 award 2/2
 (b) •¹ $\left(\frac{o}{a}\right)^2 \left(\frac{a}{h}\right)^2$
 •² $\frac{o^2 a^2}{a^2 h^2} = \frac{o^2}{h^2} = \sin^2 x$
 award 2/2

Commonly Observed Responses:

- $\frac{\cos^2 x}{\sin^2 x} \times \cos^2 x = \frac{\cos^4 x}{\sin^2 x}$ award 0/2
- $\tan^2 x(1 - \sin^2 x) = \tan^2 x - \tan^2 x \sin^2 x$ award 0/2

Question			Generic Scheme	Illustrative Scheme	Max Mark
12.	(a)		Ans: $(2x+1)(x+8)$ ¹ find an expression for the area of the rectangle	¹ $(2x+1)(x+8)$ or equivalent	1
Notes: 1. If solution to (a) appears in (b) or (c) award 1/1 2. (a) Accept $(2x+1) \times (x+8)$, $2x+1 \times x+8$ (b) Do not accept $2x+1(x+8)$, $x+8(2x+1)$ unless correct expansion appears in (a) (b) or (c)					
Commonly Observed Responses:					
12.	(b)		Ans: proof ¹ find expanded expression for area of the rectangle ² find expanded expression for area of the triangle ³ equate expanded expressions and rearrange into required form	¹ $2x^2 + 16x + x + 8$ ² $3x^2 + 15x$ ³ $2x^2 + 16x + x + 8 = 3x^2 + 15x$ $\Rightarrow x^2 - 2x - 8 = 0$	3
Notes: 1. If solution to (b) appears in (a) or (c) then all three marks are available					
Commonly Observed Responses:					

Question			Generic Scheme	Illustrative Scheme	Max Mark
12.	(c)		Ans: 12 cm and 9 cm •¹ factorise $x^2 - 2x - 8$ •² solve equation •³ reject invalid value of x and state length and breadth of rectangle	•¹ $(x-4)(x+2)$ •² $x = 4$ and $x = -2$ •³ 12 (cm) and 9 (cm)	3
Notes: 1. Correct answer without working award 0/3. 2. If solution to (c) appears in (a) or (b) then all three marks are available. 3. •¹ is available for $\frac{2 \pm \sqrt{(-2)^2 - 4 \times 1 \times (-8)}}{2 \times 1}$ 4. For an answer obtained by guess and check award 0/3					
Commonly Observed Responses: 1.(a) $(2x+1)(x+8)=0 \rightarrow x = -\frac{1}{2}$ and $x = -8$ award 1/3 $\times \checkmark \times$ (b) $x = -\frac{1}{2}$ and $x = -8$ without factorised quadratic equation stated award 0/3					

[END OF MARKING INSTRUCTIONS]